Modeling the tax policy

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Outline

1. Determining the optimal tax rate for individual
2. Determining the optimal tax rates for the state

Determining the optimal tax rate for the individual

Basics of the model

- y – labor intensity,
- x – the number of goods consumed,
- p – prices for goods,
- a – cost of living,
- r – rent from the property,
- γ – tax rate,
- d – limit of physical capabilities,
- α – propensity to consume,
- U – well-being function of the individual

Model description – 1

$$\begin{cases} \varphi(x,y) = (x-a)^{\alpha}(d-y)^{1-\alpha} \to \max \\ a \le x \le (1-\gamma)qy + r \\ 0 \le y \le d \\ 0 < \alpha < 1 \\ 0 \le \gamma \le 1 \end{cases}$$

- Because $\varphi(x,y)$ grows on x , then

$$x^{*} = (1-\gamma)qy^{*} + r$$

Model description – 2

$$a \le (1-\gamma)qy + r \Rightarrow \gamma \le 1 - \frac{a-r}{qd}$$

$$\begin{cases} \varphi(y) = ((1-\gamma)qy + r - a)^{\alpha}(d-y)^{1-\alpha} \to \max \\ \dfrac{a-r}{q} \le y \le d \\ \gamma \in \left[0; 1 - \dfrac{a}{qd}\right] \\ 0 < \alpha < 1 \end{cases}$$

$$\varphi^{'}(y) = 0 \Rightarrow y^{*} = \alpha d - \frac{(1-\alpha)(r-a)}{(1-\gamma)q} \Rightarrow$$

$$x^{*} = (1-\gamma)\alpha qd - (1-\alpha)(r-a) + r$$

Model

- There are N individuals living in the state, each of whom determines his work activity according to a model

$$\begin{cases} \varphi(x,y) = (x-a)^{\alpha}(d-y)^{1-\alpha} \to \max \\ a \le x \le (1-\gamma)qy + r \\ 0 \le y \le d \\ 0 < \alpha < 1 \\ 0 \le \gamma \le 1 \end{cases}$$

State tax revenues

$$\begin{cases} \varphi(\gamma) = N\gamma y(\gamma) q \to \max \\ 0 \le \gamma \le 1 \end{cases}$$

- $\gamma = 0$ – The general revenues will be 0.
- If $\gamma = 1$, all individuals must refuse to work legally, so the general revenues goes to 0.
- Consider the collection of taxes depending on the tax rate, i.e. $0 \le \gamma \le 1$

Collection of taxes depending on the tax rate – 1

$$\varphi(\gamma) = N\gamma\left(\alpha d - \frac{(1-\alpha)(r-a)}{(1-\gamma)q}\right)q \to \max$$

$$\gamma = 0, \varphi(\gamma) = 0$$

$$\gamma = 1, \varphi(\gamma) = -\infty$$

$$\varphi(\gamma) = 0 \Rightarrow \gamma^0 = 1 - \frac{(1-\alpha)(r-a)}{\alpha d q}$$

$$0 \le \gamma \le \gamma^0$$

Collection of taxes depending on the tax rate – 2

$$\varphi'(\gamma) = 0$$

$$\alpha d - \frac{(1-\alpha)(r-a)}{(1-\gamma)^2 q} = 0$$

$$\gamma^* = 1 - \sqrt{\frac{(1-\alpha)(r-a)}{\alpha d q}}$$

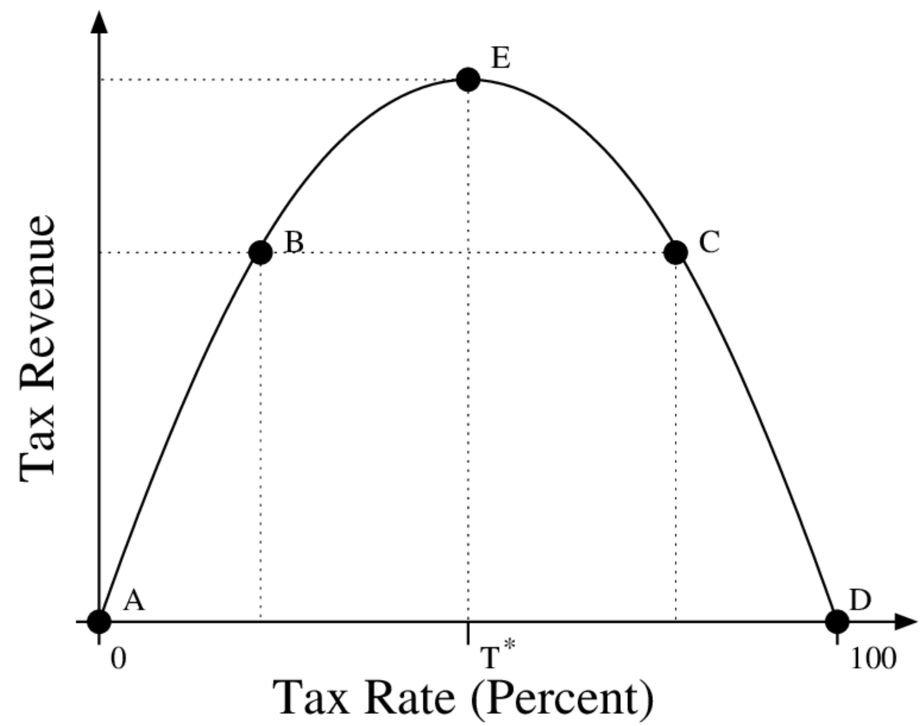
$$1 - \gamma^* = \sqrt{1 - \gamma^0}$$

Conclusions of the model

- increase of α increases tax revenues;
- improving human health (d) increases tax revenues;
- increase of a leads to an increase in tax revenues;
- an increase in wages increases state revenues.

Laffer curve

- Laffer curve – geometric location of points, which characterizes the dependence of government revenues on the average level of tax rates in the country.



Example

- The population of the state is 50 million people.
- tendency to rest, $1-\alpha=0,7$
- subsistence level – USD 300,
- limit of physical capabilities 250 hours,
- salary 10 USD/hour,
- rental income USD 20 / day.

Determine the optimal tax rate and tax revenues of the state, build a Laffer curve.

Solution – 1

$$\varphi(\gamma) = N\gamma\left(\alpha d - \frac{(1-\alpha)(r-a)}{(1-\gamma)q}\right)q \to \max,$$

$$0 \le \gamma \le 1$$

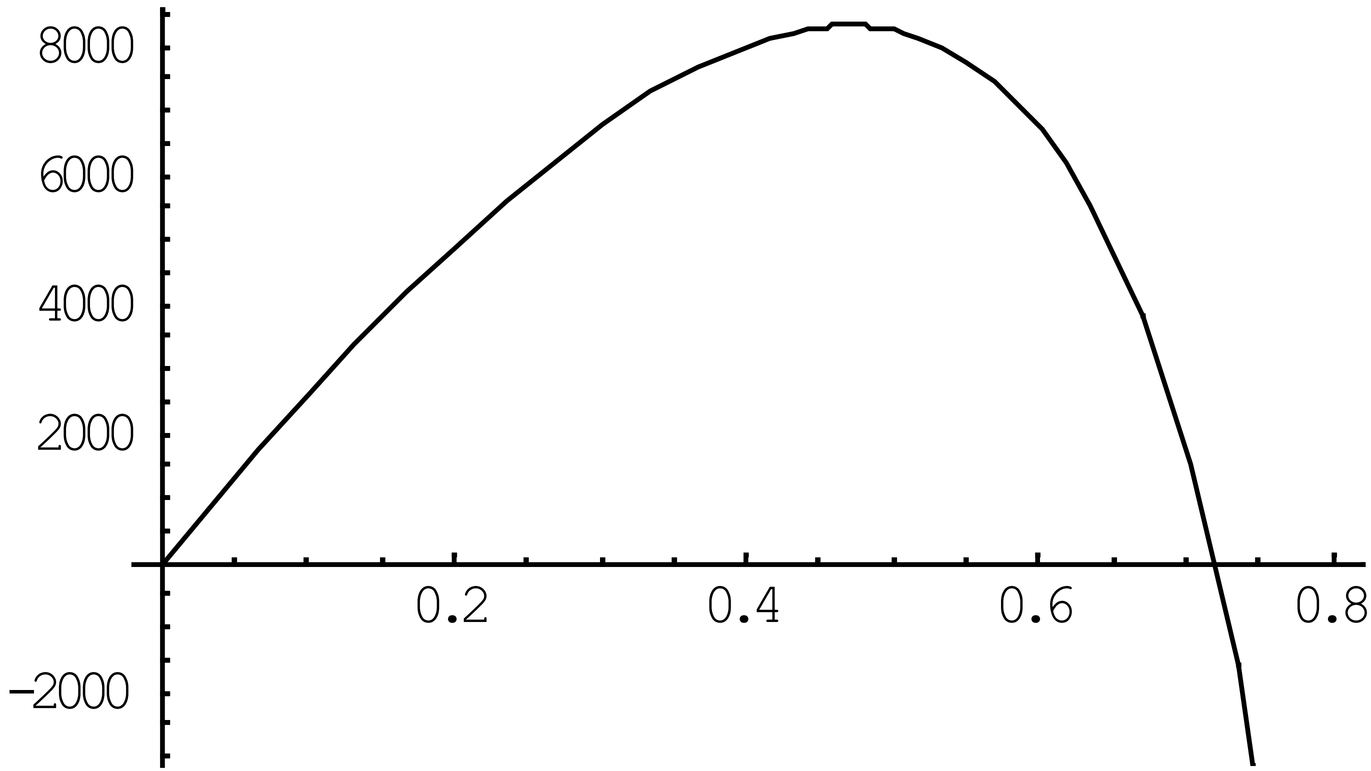
$$\varphi(\gamma) = 50\gamma\left(0,3\cdot 250 - \frac{(0,7)(20\cdot 30 - 300)}{(1-\gamma)10}\right)\cdot 10 = 500\gamma\left(75 - \frac{21}{1-\gamma}\right) \to \max,$$

$$0 \le \gamma \le 1$$

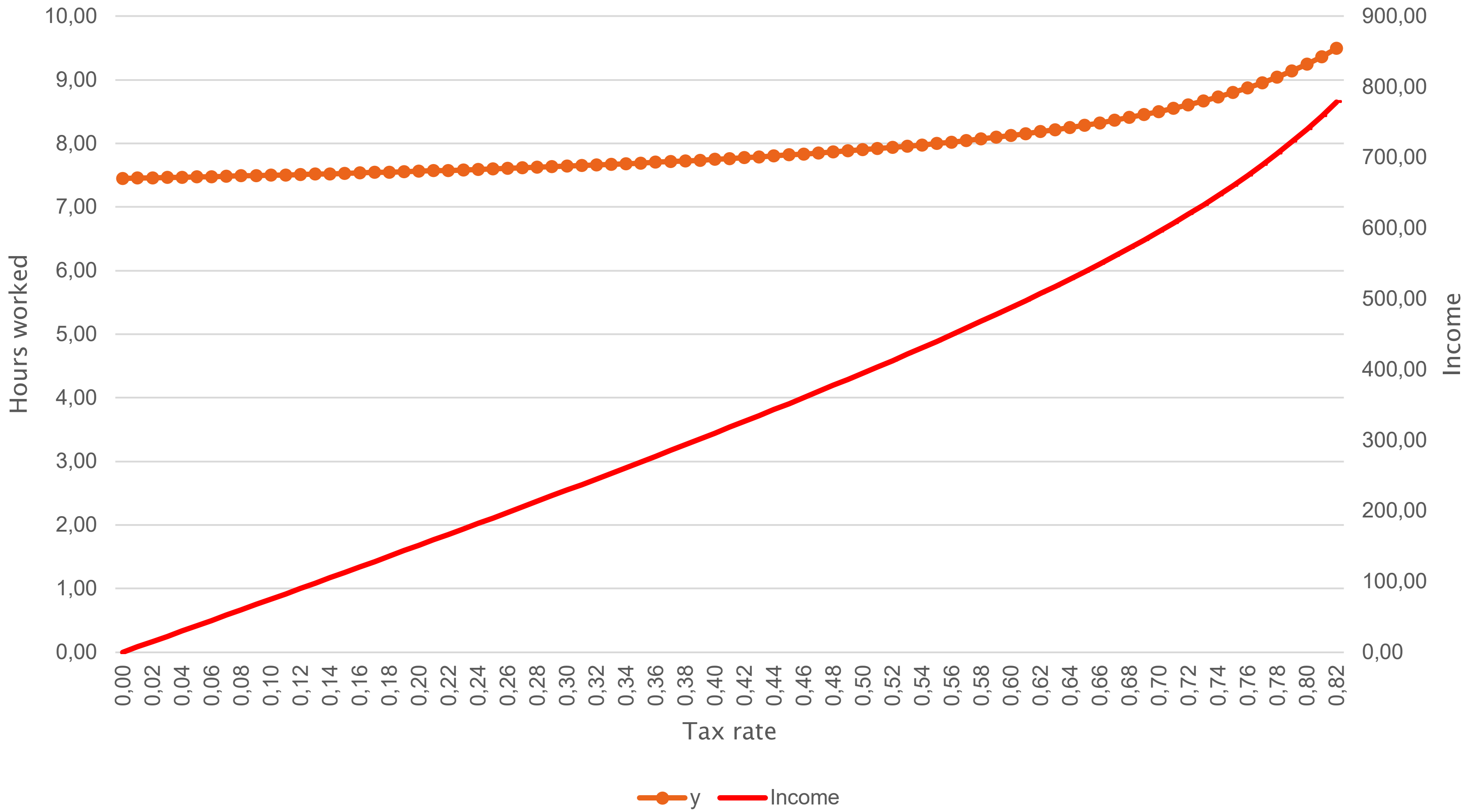
$$\gamma^{*} = 1 - \sqrt{\frac{(1-\alpha)(r-a)}{\alpha d q}} = 1 - \sqrt{\frac{0,7\cdot 300}{0,3\cdot 250\cdot 10}} = 0,47085$$

$$\gamma^{0} = 1 - \frac{(1-\alpha)(r-a)}{\alpha d q} = 1 - \frac{0,7\cdot 300}{0,3\cdot 250\cdot 10} = 0,72$$

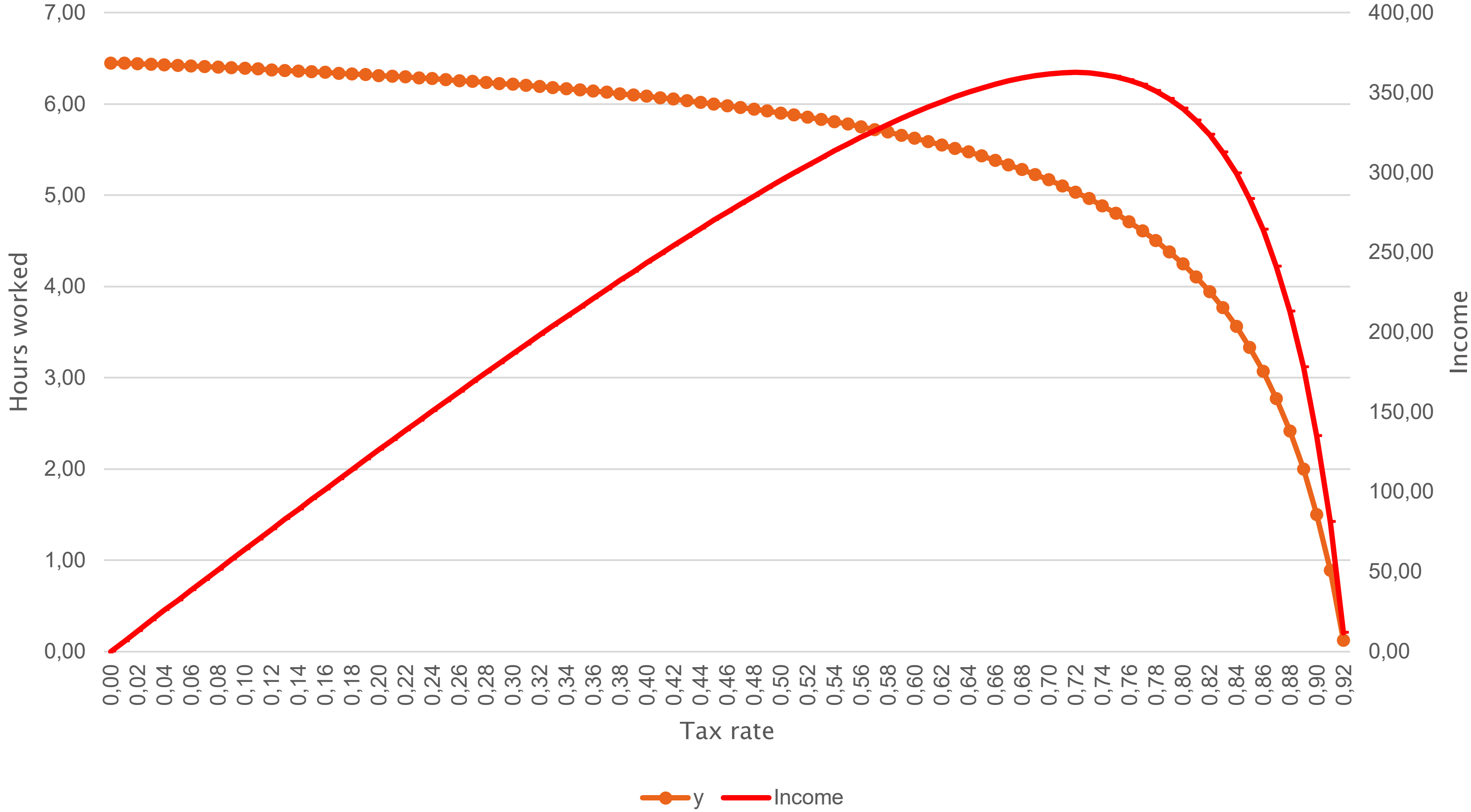
Solution – 2



Dependence of an individual's labor activity on the tax rate in the absence of additional income



Dependence of an individual's labor activity on the tax rate in the case of additional income

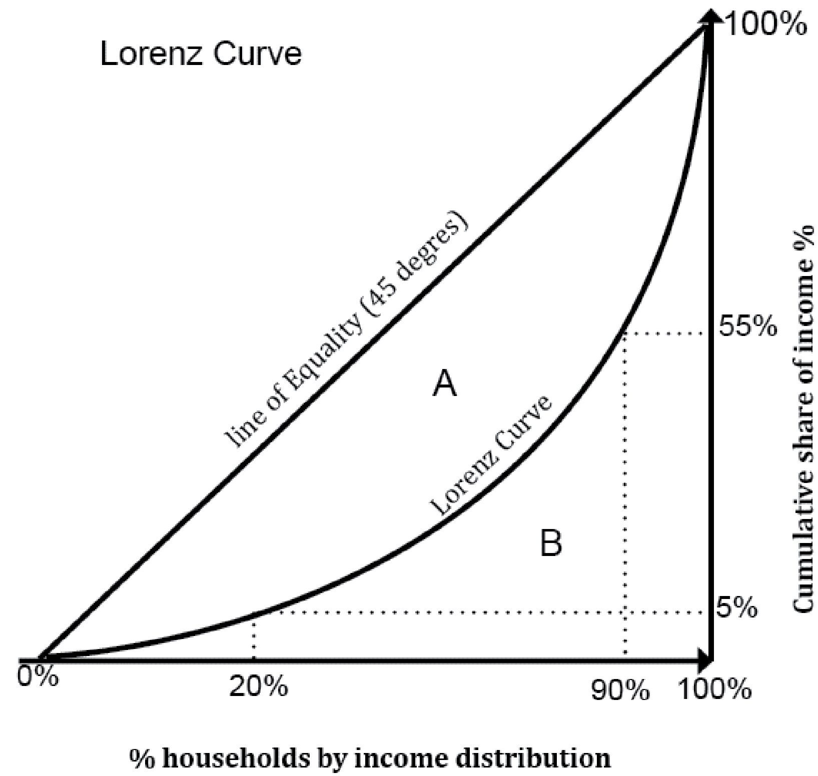


The problem of income inequality

- The 2016 Davos Forum notes that income inequality results in the prevailing ideology of populism, which in turn leads to growing economic imbalances, military clashes and the formation of totalitarian regimes.
- Among the Organization for Economic Co-operation and Development, the lowest levels of inequality were recorded in Denmark, Slovakia and Slovenia, and the highest in Mexico, Turkey and the United States.
- However, about a quarter of the EU population was at risk of poverty (at risk of poverty, according to Eurostat, means a state in which residents have a weighted average per capita income of less than 60% of the median median income available nationwide). such persons grows by about 0.2% each year. 17% of the EU-28 population is at risk of income poverty, ie the amount of their income after paying taxes and receiving social transfers was below the subsistence level.
- Researchers note that it is impossible to predict economic growth without considering the factor of economic inequality, because in the medium term, the growth of inequality helps to slow down economic growth. It is worth noting that income inequality cannot be overcome quickly, although some economic growth can be achieved in the short term.

Lorenz curve

- Lorenz curve – Characterize sdegree of equality (inequality) in income distribution.



Ginny coefficient

- **Ginny coefficient** – indicator of inequality distribution, accepts value between 0 and 1,

where

- 0 means absolute equality,
- 1 indicates full inequality.

How to solve the problem of inequality?

Suggestions for a solution

- increase in property taxes;
- progressive personal income taxes;
- introduction of inheritance taxes and property gifts;
- expanding the access of poor families to education and health care;
- increasing the targeting of social benefits;
- increasing incentives to work.

The concept of unconditional income

- Some governments have begun to consider the so-called concept of unconditional basic income, according to which each member of the community is regularly paid a certain amount of money, regardless of income and employment.
- The main goal of introducing insane income is, of course, to solve the problem of poverty, economic inequality, technological unemployment, as well as a significant reduction in the administration of social programs, the abolition of numerical inspections, the elimination of corruption in the distribution of social benefits.
- Another advantage of such an approach should be a freer choice of a person's workplace, and hence a more flexible adaptation of society to market conditions, greater efficiency.

Analysis

- Analysis of the model shows that the availability of additional income not only contributes to more effective involvement of individuals in work, but is also an indisputable factor in guaranteeing the freedom of individuals, increasing mutual respect between them.

Programs UI in different countries

- The EU has developed an anti-poverty program to implement the UI concept. EUR 13 million has been allocated for the project.
- The experiment was conducted in Barcelona (Spain), Utrecht (Netherlands) and Helsinki (Finland) during 2017-2018.
- The payment was 400-525 euros per month.
- Four groups of participants received assistance in different ways.

Current experiments

- Social income began to be paid in the form of mobile money to people in need in Sierra Leone. The international initiative is funded by contributions from people around the world who donate 1% of their monthly salaries.
- In May 2020, Spain introduced a mi nimum basic income of about 2% of the population in response toCOVID-19,to "fight the jump in poverty through a pandemic coronavirus. It is expected to spend three billion euros a year.
- In August 2020, a project was launched in Germany that provides citizens who apply online with 1,200 euros of monthly basic income from the lottery system.
- In October 2020, the project was launched in Hudson, New York. He allocates $ 500 monthly basic income to 25 residents. This will last for five years and will be compared to 50 people who do not receive a basic income.

Poll

According to opinion polls in Switzerland:

- about 90% of respondents will continue to work at the same job,
- 8% will change jobs,
- 2% will not work if the new system is introduced.

However, these data are only the opinion of respondents, whether they will correspond to reality, only time will tell.

Will you work under unconditional income?

Determining the optimal tax rates for the state

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Model assumption

- The volume of GDP *X*, depends on the level of tax burden

$$\theta = \frac{T}{X}$$

where *T* – the amount of tax revenues to budget of the country.

- The dependence $X(\theta)$ can be approximated by some nonlinear function, the parameters of which are subject to quantification.

Laffer point of first type

- Laffer point of first type θ^* is called the tax rate at which the production curve $X(\theta)$ reaches a local maximum, ie when the conditions are met:

$$\frac{\partial X(\theta^*)}{\partial \theta^*} = 0$$

$$\frac{\partial^2 X(\theta^*)}{\partial (\theta^*)^2} < 0$$

Laffer point of second type

- Laffer point of second type θ^{**} is called the tax rate at which the fiscal curve $T = T(\theta)$ reaches a local maximum, i.e. when the conditions are met:

$$\frac{\partial T(\theta^{**})}{\partial \theta^{**}} = 0$$

$$\frac{\partial T^2(\theta^{**})}{\partial (\theta^{**})^2} < 0$$

Model analysis

- If $\theta < \theta^{*}$, the state stimulates the overall further development of the economy, without the aim of maximizing taxes or maximizing gross product.
- If $\theta^{*} < \theta < \theta^{**}$, the tax burden is quite normal.
- If $\theta > \theta^{**}$, the state pursues an extremely ill-considered fiscal policy, which threatens a long-term recession and the development of the shadow economy.

Polynomial interpolation

$$X = \sum_{i=0}^{m} \beta_i \theta^i$$

- where β_i – the parameters to be statistically estimated on the basis of regression analysis.

$$T = \theta \cdot X$$

$$T = \sum_{i=0}^{m} \beta_i \theta^{i+1}$$

Example

$$X = \beta_0 + \beta_1\theta + \beta_2\theta^2$$

$$\theta^* = -\frac{\beta_1}{2\beta_2}$$

$$\theta^{**} = -\frac{\beta_1 + \sqrt{\beta_1^2 - 3\beta_0\beta_2}}{3\beta_2}$$

$$X_{\text{max}} = X\left(\theta^*\right) = \beta_0 - \frac{\beta_1^2}{4\beta_2}$$

$$T\left(\theta^*\right) = \frac{\beta_1^3 - 4\beta_0\beta_1\beta_2}{8\beta_2^2}$$

$$X\left(\theta^{**}\right) = \frac{6\beta_0\beta_2 - \beta_1\left(\beta_1 + \sqrt{\beta_1^2 - 3\beta_0\beta_2}\right)}{9\beta_2}$$

$$T_{\text{max}} = T\left(\theta^{**}\right) = \frac{\left(\beta_1 + \sqrt{\beta_1^2 - 3\beta_0\beta_2}\right)\left(-6\beta_0\beta_2 + \beta_1\left(\beta_1 + \sqrt{\beta_1^2 - 3\beta_0\beta_2}\right)\right)}{27\beta_2^2}$$

Power function

$$T(\theta) = \lambda \theta^{\alpha} (1 - \theta)^{\beta}$$

- α – is the coefficient of tax progression, which is legally enshrined in the tax system,
- β – is the coefficient of sensitivity of the economy to changes in the tax rate (coefficient of economic activity).
- λ – coefficient of proportionality.

Example

$$X\left(\theta\right)=\frac{T\left(\theta\right)}{\theta}=\lambda\theta^{\alpha-1}\left(1-\theta\right)^{\beta}$$

$$\theta^{*}=\frac{1-\alpha}{1-\alpha-\beta}$$

$$\theta^{**}=\frac{\alpha}{\alpha+\beta}$$

Production and institutional function

$$X = \gamma D(t) K^{(\alpha+\beta\theta)\theta} L^{(n+m\theta)\theta}$$

- X – GDP,
- K – capital of the country (volume of fixed assets),
- L – labor (number of employees employed in the economy),
- θ – tax burden on the country's economy,
- $D(t)$ – trend operator ($D(t) = e^{\lambda t}$),
- γ, α, β, n, m – model coefficients.

Example

$$X = \gamma D(t) K^{(\alpha+\beta\theta)\theta} L^{(n+m\theta)\theta}$$

$$\theta^{*} = -\frac{1}{2}\frac{a \ln K + n \ln L}{b \ln K + m \ln L}$$

$$\theta^{**} = \frac{1}{4}\frac{\pm\sqrt{(a \ln K + n \ln L)^{2} - 8(b \ln K + m \ln L)} - a \ln K - n \ln L}{b \ln K + m \ln L}$$

Laffer points for VAT

Year	Laffer point of first type	Laffer point of second type
1999	0.046411	0.079421
2000	0.052804	0.084871
2001	0.057881	0.089265
2002	0.058156	0.089496
2003	0.062661	0.093421
2004	0.066539	0.096824
2005	0.072211	0.101832
2006	0.078201	0.107196
2007	0.086753	0.114929
2008	0.092212	0.119912
2009	0.084096	0.112560
2010	0.086100	0.114376
2011	0.091283	0.119103
2012	0.094988	0.122500
2013	0.092962	0.120636
2014	0.090094	0.118012

Laffer points for PIT

Year	Laffer point of first type	Laffer point of second type
1999	0.034209	0.050277
2000	0.036975	0.051796
2001	0.038859	0.052866
2002	0.038955	0.052916
2003	0.040429	0.053765
2004	0.041578	0.054435
2005	0.043091	0.055326
2006	0.044503	0.056182
2007	0.046252	0.057258
2008	0.047232	0.057871
2009	0.045739	0.056958
2010	0.046128	0.057198
2011	0.047072	0.057787
2012	0.047696	0.058180
2013	0.047360	0.057967
2014	0.046863	0.057655

Laffer points for profits of enterprises

Year	Laffer point of first type	Laffer point of second type
1999	0.018895	0.060770
2000	0.027296	0.063192
2001	0.032446	0.064829
2002	0.032695	0.064903
2003	0.036428	0.066154
2004	0.039183	0.067116
2005	0.042619	0.068355
2006	0.045647	0.069513
2007	0.049177	0.070917
2008	0.051057	0.071692
2009	0.048165	0.070540
2010	0.048935	0.070848
2011	0.050755	0.071596
2012	0.051924	0.072085
2013	0.051297	0.071819
2014	0.050358	0.071428

Year	VAT tax burden on GDP	Laffer point of 1st type	Laffer point of 2st type	Tax burden of personal income tax on GDP	Laffer point of 1st type	Laffer point of 2st type	Income tax burden on GDP	Laffer point of 1st type	Laffer point of 2st type
1995	0.03	0.07	0.15	0.01	0.01	0.39	0.01	0.03	0.07
1996	0.04	0.07	0.16	0.01	-0.01	0.42	0.02	0.03	0.07
1997	0.09	0.07	0.16	0.02	-0.07	0.44	0.04	0.03	0.07
1998	0.09	0.06	0.17	0.03	-0.59	0.46	0.05	0.03	0.07
1999	0.06	0.06	0.18	0.03	0.17	0.49	0.05	0.03	0.07
2000	0.06	0.06	0.19	0.04	0.12	0.51	0.05	0.03	0.07
2001	0.05	0.06	0.19	0.04	0.11	0.52	0.04	0.03	0.07
2002	0.06	0.06	0.19	0.05	0.11	0.52	0.04	0.03	0.07
2003	0.05	0.06	0.20	0.05	0.10	0.53	0.05	0.03	0.07
2004	0.08	0.06	0.20	0.04	0.10	0.54	0.05	0.02	0.07
2005	0.11	0.06	0.20	0.04	0.09	0.55	0.05	0.02	0.07
2006	0.12	0.05	0.21	0.04	0.09	0.57	0.05	0.01	0.07
2007	0.11	0.05	0.22	0.05	0.08	0.60	0.05	0.01	0.07
2008	0.13	0.03	0.24	0.05	0.07	0.65	0.05	0.06	0.07
2009	0.13	0.01	0.26	0.05	0.07	0.69	0.04	0.05	0.07
2010	0.12	0.25	0.29	0.05	0.07	0.79	0.04	0.04	0.07
2011	0.13	0.18	0.30	0.05	0.07	0.81	0.04	0.04	0.07
2012	0.13	0.13	0.31	0.05	0.07	0.85	0.04	0.04	0.07
2013	0.12	0.12	0.32	0.05	0.07	0.88	0.04	0.04	0.07
2014	0.12	0.11	0.35	0.05	0.06	0.96	0.03	0.04	0.07
2015	0.12	0.17	0.30	0.05	0.07	0.81	0.02	0.04	0.07
2016	0.16	0.14	0.31	0.07	0.07	0.85	0.03	0.04	0.07

Taxation of production factors – 1

- Let the government have two tax bases:
  - taxes on capital gains K;
  - labor income taxes L.

- Each of the tax bases in the period depends on tax rates:

$K(\theta_t)$, $L(\tau_t)$, where θ_t and τ_t – tax rates on capital and labor income, respectively.

- $K(\theta_t)$ and $L(\tau_t)$ – descending functions.

Taxation of production factors – 2

- Government revenues:

$$G = \theta_t K(\theta_t) + \tau_t L(\tau_t)$$

- Private agents decide on capital inflows during the period t–1;
- the decision on the supply of labor is made in the period t.

Example – 1

- Determine the optimal tax rates on labor and capital and find the budget revenue provided that:

$K(\theta_t) = 5000(3 - 5 * \theta_t + 2 * \theta_t^2)$ and $L(\tau_t) = 4000(1 - \tau_t)$

Solution

- We maximize tax revenues from labor

$$4000(\tau_t - \tau_t^2) \to \max \quad \left(4000(\tau_t - \tau_t^2)\right)' = 1 - 2\tau_t = 0\,; \tau_t^* = 0{,}5$$

- We maximize revenues on capital

$$5000(3\theta_t - 5\theta_t^2 + 2\theta_t^3) \to max$$

$$\left(5000(3\theta_t - 5\theta_t^2 + 2\theta_t^3)\right)' = 3 - 10\theta_t + 6\theta_t^2 = 0$$

$$\theta_{t1} = \frac{10 - \sqrt{28}}{12} = 0{,}39 \qquad \theta_{t2} = \frac{10 + \sqrt{28}}{12} = 1{,}27$$

$$G^* = 5000(1{,}17 - 0{,}7605 + 0{,}1186) + 4000(0{,}25) = 3640{,}5$$

Thank you!